EX 28 :

$f(x)=\dfrac{4^x}{4^{2x}-1}$; $x\in\mathbb{R}^*$

1) $f(x)=\dfrac{4^x}{4^x(4^x-4^{-x})}=\dfrac{1}{4^x-4^{-x}}$

pour $x\in D_f=IR^*$ on a $(-x)\in D_f$

$f(-x)=\dfrac{1}{4^{-x}-4^x}=\dfrac{-1}{4^x-4^{-x}}=-f(x)$

d'où f est impaire

2) $f'(x)=\dfrac{4^x.\ln 4\left(4^{2x}-1\right)-4^x\left(2\ln 4\times 4^{2x}\right)}{\left(4^{2x}-1\right)^2}$

$f'(x)=\dfrac{(\ln 4).4^x.\left[4^{2x}-1-2\times 4^{2x}\right]}{\left(4^{2x}-1\right)^2}$

$f'(x)=\dfrac{(\ln 4).4^x.\left(-4^{2x}-1\right)}{\left(4^{2x}-1\right)^2}$

$f'(x)<0$; $\forall x\in\ ]0,+\infty[$

x	0		$+\infty$
f '(x)	||	-	
f(x)	$+\infty$	↘	0

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\dfrac{4^x}{4^{2x}-1}=+\infty$

$\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{1}{4^x-\left(\frac{1}{4}\right)^x}=0$

3) a) $f(x)=\dfrac{4}{15}\Leftrightarrow\dfrac{4^x}{4^{2x}-1}=\dfrac{4}{15}$

$\Leftrightarrow 4.(4^{2x}-1)=15\times 4^x\Leftrightarrow 4.(4^{2x})-15\times 4^x-4=0$

on pose $t=4^x$

l'équation devient : $4t^2-15t-4=0$

$\Delta=289\to\sqrt{\Delta}=17$

$t'=\dfrac{1}{4}$; $t''=4$

$t=\dfrac{1}{4}\Leftrightarrow 4^x=\dfrac{-1}{4}$ impossible

$t=4\Leftrightarrow 4^x=4\Leftrightarrow x=1$

$S_{IR}=\{1\}$

b) $f(x)=\dfrac{-4}{15}\Leftrightarrow f(-x)=\dfrac{4}{15}$ car f est impaire

$\Leftrightarrow -x=1\ \Leftrightarrow x=-1$

$S_{IR}=\{-1\}$

EX 29 :

1) $\int_0^1 3^x dx=\int_0^1 e^{x\ln 3}dx=\left[\dfrac{1}{\ln 3}e^{x\ln 3}\right]_0^1=\left[\dfrac{1}{\ln 3}(3^x)\right]_0^1=\dfrac{2}{\ln 3}$

2) $\int_0^{-1}\dfrac{3^x}{1+3^x}dx=\int_0^{-1}\dfrac{1}{\ln 3}.\dfrac{(\ln 3).3^x}{1+3^x}dx=\dfrac{1}{\ln 3}\left[\ln(1+3^x)\right]_0^{-1}$

$=\dfrac{1}{\ln 3}\left[\ln\left(\dfrac{4}{3}\right)-\ln(2)\right]=\dfrac{\ln(\frac{2}{3})}{\ln 3}=\dfrac{\ln 2}{\ln 3}-1$

3) $\int_0^{\frac{1}{\ln 2}}2^x.(1+2^x)^2dx=\left[\dfrac{1}{\ln 2}\left(\dfrac{1}{3}(1+2^x)\right)^3\right]_0^{\frac{1}{\ln 2}}$

$=\dfrac{1}{3\ln 2}\left[(1+2^x)^3\right]_0^{\frac{1}{\ln 2}}=\dfrac{1}{3\ln 2}\left[(1+e)^3-2^3\right]$

car $2^{\frac{1}{\ln 2}}=e$

4) $\int_1^2 x^{\frac{4}{3}}dx=\left[\dfrac{1}{\frac{4}{3}+1}x^{\frac{4}{3}+1}\right]_1^2=\left[\dfrac{3}{7}x^{\frac{7}{3}}\right]_1^2=\dfrac{3}{7}(2^{\frac{7}{3}}-1)$

5) $\int_{\frac{1}{2}}^1 4.x^{\frac{-1}{5}}dx=\left[5x^{\frac{4}{5}}\right]_{\frac{1}{2}}^1=5(1-2^{\frac{-4}{5}})$

6) $\int_0^1\sqrt[4]{x}dx=\int_0^1 x^{\frac{1}{4}}dx=\left[\dfrac{4}{5}x^{\frac{5}{4}}\right]_0^1=\dfrac{4}{5}$

7) $\int_{\frac{1}{2}}^1(3x-1)^{\frac{3}{2}}dx=\dfrac{1}{3}\int_{\frac{1}{2}}^1 3(3x-1)^{\frac{3}{2}}dx=\dfrac{1}{3}\left[\dfrac{2}{5}(3x-1)^{\frac{5}{2}}\right]_{\frac{1}{2}}^1$

$=\dfrac{2}{15}(2^{\frac{5}{2}}-2^{\frac{-5}{2}})=\dfrac{2}{15}(4\sqrt{2}-\dfrac{1}{4\sqrt{2}})=\dfrac{31}{30\sqrt{2}}$

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